Please take your time and do a good job working through these problems on loose leaf paper.

Once you have completed, please check your answers with the answers at the end.

ANGLE PAIR RELATIONSHIPS   8.3.2

Properties of Angle Pairs

Intersecting lines form four angles. The pairs of angles across from each other are called vertical angles. The measures of vertical angles are equal.
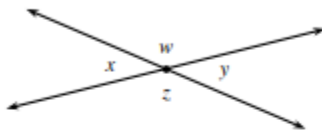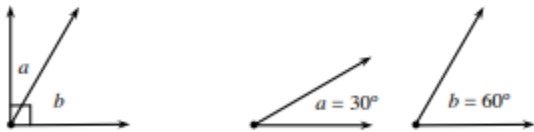
$\angle x$ and $\angle y$ are vertical angles
$\angle w$ and $\angle z$ are vertical angles

If the sum of the measures of two angles is exactly 180°, then the angles are called supplementary angles.

$c = 110°$ $d = 70°$
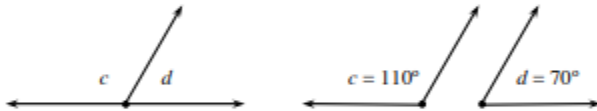
$\angle c$ and $\angle d$ are supplementary angles

If the sum of the measures of two angles is exactly 90°, then the angles are called complementary angles.

$a = 30°$ $b = 60°$

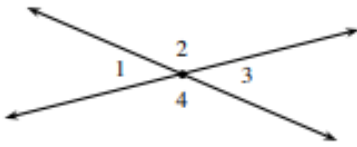$\angle a$ and $\angle b$ are complementary angles

Angles that share a vertex and one side but have no common interior points (that is, do not overlap each other) are called adjacent angles.
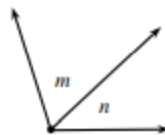
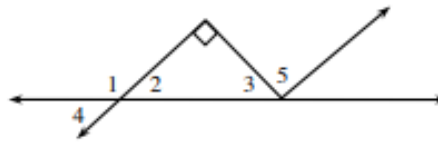$\angle m$ and $\angle n$ are adjacent angles

Example 1

Find the measure of the missing angles if $m\angle 3 = 50°$.

- $m\angle 1 = m\angle 3$ (vertical angles)
  $\Rightarrow m\angle 1 = 50°$

- $\angle 2$ and $\angle 3$ (supplementary angles)
  $\Rightarrow m\angle 2 = 180° - 50° = 130°$

- $m\angle 2 = m\angle 4$ (vertical angles)
  $\Rightarrow m\angle 4 = 130°$

Example 2

Classify each pair of angles below as vertical, supplementary, complementary, or adjacent.

a. $\angle 1$ and $\angle 2$ are adjacent and supplementary

b. $\angle 2$ and $\angle 3$ are complementary

c. $\angle 3$ and $\angle 5$ are adjacent

d. $\angle 1$ and $\angle 4$ are adjacent and supplementary

e. $\angle 2$ and $\angle 4$ are vertical

Problems

Find the measure of each angle labeled with a variable.

1.

2.

3.

4.

5.

6.
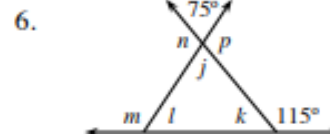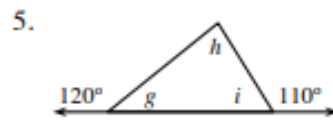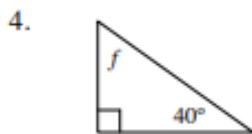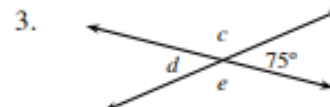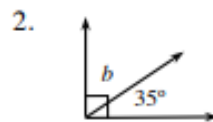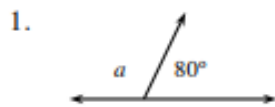
AREA OF A CIRCLE

In class, students have done explorations with circles and circular objects to discover the relationship between circumference, diameter, and pi (π). To read more about the in-class exploration of area, see problems 9-22 through 9-26 (especially 9-26) in the *Core Connections, Course 2* text.

In order to find the area of a circle, students need to identify the radius of the circle. The radius is half the diameter. Next they will square the radius and multiply the result by π. Depending on the teacher's or book's preference, students may use $\frac{22}{7}$ for π when the radius or diameter is a fraction, 3.14 for π as an approximation, or the π button on a calculator. When using the π button, most teachers will want students to round to the nearest tenth or hundredth.

The formula for the area of a circle is: $A = r^2 \pi$.

Example 1

Find the area of a circle with $r = 17$ feet.

$A = (17)^2 \pi$
$ = (17 \cdot 17)(3.14)$
$ = 907.46$ square feet

Example 2

Find the area of a circle with $d = 84$ cm.

$r = 42$ cm
$A = (42)^2 \pi$
$ = (42 \cdot 42)(3.14)$
$ = 5538.96$ square cm

Problems

Find the areas of the circles with the following radii or diameter lengths. Use 3.14 for the value of π. Round to the nearest hundredth.

1. $r = 6$ cm
2. $r = 3.2$ in.
3. $d = 16$ ft
4. $r = \frac{1}{2}$ m

SURFACE AREA OF A PRISM

The surface area of a prism is the sum of the areas of all of the faces, including the bases. Surface area is expressed in square units.

For additional information, see the Math Notes box in Lesson 9.2.4 of the *Core Connections, Course 2* text.

Example

Find the surface area of the triangular prism at right.

Step 1: Area of the 2 bases: $2\left[\frac{1}{2}(6 \text{ cm})(8 \text{ cm})\right] = 48 \text{ cm}^2$

Step 2: Area of the 3 lateral faces

Area of face 1: $(6 \text{ cm})(7 \text{ cm}) = 42 \text{ cm}^2$
Area of face 2: $(8 \text{ cm})(7 \text{ cm}) = 56 \text{ cm}^2$
Area of face 3: $(10 \text{ cm})(7 \text{ cm}) = 70 \text{ cm}^2$

Step 3: Surface Area of Prism = sum of area of bases and area of the lateral faces:
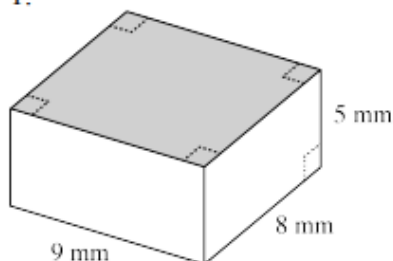
$$SA = 48 \text{ cm}^2 + 42 \text{ cm}^2 + 56 \text{ cm}^2 + 70 \text{ cm}^2 = 216 \text{ cm}^2$$
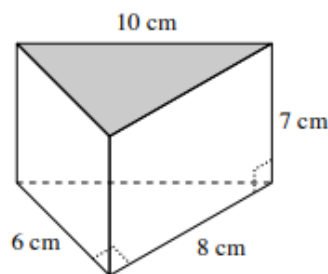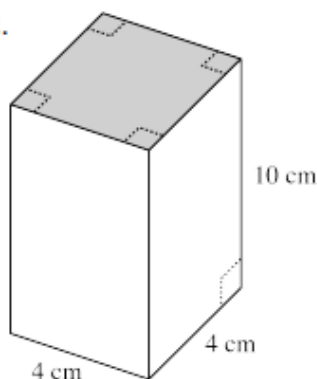
Problems

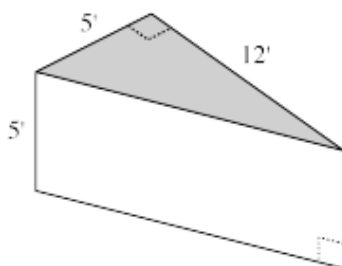Find the surface area of each prism.

1.

5 mm

8 mm

9 mm

2.

10 cm

4 cm

4 cm

3.

5'

12'

5'

VOLUME OF A PRISM

Volume is a three-dimensional concept. It measures the amount of interior space of a three-dimensional figure based on a cubic unit, that is, the number of 1 by 1 by 1 cubes that will fit inside a figure.
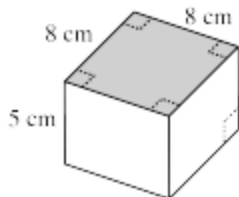
The volume of a prism is the area of either base (B) multiplied by the height (h) of the prism.

$$V = \text{(Area of base)} \cdot \text{(height)} \quad \text{or} \quad V = Bh$$

For additional information, see the Math Notes box in Lesson 9.2.4 of the *Core Connections, Course 2* text.

Example 1

Find the volume of the square prism below.

The base is a square with area (B)
$8 \cdot 8 = 64$ units^2.

$$\begin{aligned} \text{Volume} &= B(h) \\ &= 64(5) \\ &= 320 \text{ units}^3 \end{aligned}$$

Example 2

Find the volume of the triangular prism below.
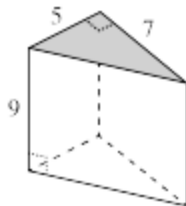
The base is a right triangle with area
$\frac{1}{2}(5)(7) = 17.5$ units^2.

$$\begin{aligned} \text{Volume} &= B(h) \\ &= 17.5(9) \\ &= 157.5 \text{ units}^3 \end{aligned}$$

Example 3

Find the volume of the trapezoidal prism below.
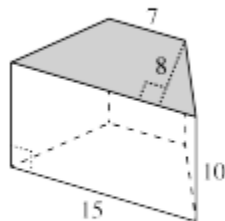
The base is a trapezoid with area $\frac{1}{2}(7+15)\cdot 8 = 88$ units^2.

$$\begin{aligned} \text{Volume} &= B(h) \\ &= 88(10) \\ &= 880 \text{ units}^3 \end{aligned}$$

Example 4

Find the height of a right prism with a volume of 132.5 cm^3 and base area of 25 cm^2.

$$\begin{aligned} \text{Volume} &= B(h) \\ 132.5 &= 25(h) \\ h &= \frac{132.5}{25} \\ h &= 5.3 \text{ cm} \end{aligned}$$

Problems

Calculate the volume of each prism. The base of each figure is shaded.
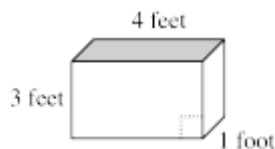
1. Rectangular Prism   2. Right Triangular Prism   3. Rectangular Prism

4 feet

3 feet

1 foot

6 cm

8 cm

7 cm

5 in.   6 in.

8.5 in.
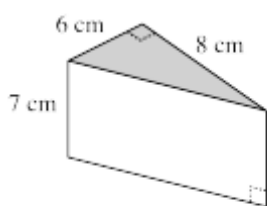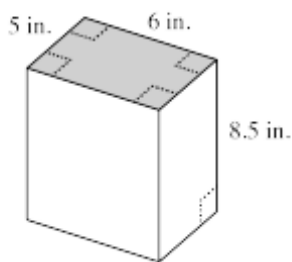
<table>
<tr><td colspan="2">MEASURES OF CENTRAL TENDENCY</td><td>8.1.1 and 8.1.2</td></tr>
</table>

Measures of central tendency are numbers that locate or approximate the "center" of a set of data—that is, a "typical" value that describes the set of data. Mean and median are the most common measures of central tendency.

The mean is the arithmetic average of a data set. Add all the values in a set and divide this sum by the number of values in the set. The median is the middle number in a set of data arranged numerically. An outlier is a number that is much smaller or larger than most of the others in the data set. The range of a data set is the difference between the highest and lowest values of the data set.

For additional information, see the Math Notes box in Lesson 8.1.2 of the *Core Connections, Course 1* text.

The mean is calculated by finding the sum of the data set and dividing it by the number of elements in the set.

Example 1

Find the mean of this set of data: 34, 31, 37, 44, 38, 34, 42, 34, 43, and 41.

- $34 + 31 + 37 + 44 + 38 + 34 + 42 + 34 + 43 + 41 = 378$

- $\frac{378}{10} = 37.8$

The mean of this set of data is 37.8.

Example 2

Find the mean of this set of data: 92, 82, 80, 92, 78, 75, 95, and 77.

- $92 + 82 + 80 + 92 + 78 + 75 + 95 + 77 = 671$

- $\frac{671}{8} = 83.875$

The mean of this set of data is 83.875.

Problems

Find the mean of each set of data.

1. 29, 28, 34, 30, 33, 26, and 34.

2. 25, 34, 35, 27, 31, and 30.

3. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.

4. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

The median is the middle number in a set of data *arranged in numerical order*. If there is an even number of values, the median is the mean (average) of the two middle numbers.

Example 3

Find the median of this set of data: 34, 31, 37, 44, 38, 34, 43, and 41.

*   Arrange the data in order: 31, 34, 34, 37, 38, 41, 43, 44.

*   Find the middle value(s): 37 and 38.

*   Since there are two middle values, find their mean: $37 + 38 = 75$, $\frac{75}{2} = 37.5$. Therefore, the median of this data set is 37.5.

Example 4

Find the median of this set of data: 92, 82, 80, 92, 78, 75, 95, 77, and 77.

*   Arrange the data in order: 75, 77, 77, 78, 80, 82, 92, 92, 95.

*   Find the middle value(s): 80. Therefore, the median of this data set is 80.

Problems

Find median of each set of data.

5. 29, 28, 34, 30, 33, 26, and 34.

6. 25, 34, 27, 25, 31, and 30.

7. 80, 89, 79, 84, 95, 79, 78, 89, 76, 82, 76, 92, 89, 81, and 123.

8. 116, 104, 101, 111, 100, 107, 113, 118, 113, 101, 108, 109, 105, 103, and 91.

> The range of a set of data is the difference between the highest value and the lowest value.

Example 5

Find the range of this set of data: 114, 109, 131, 96, 140, and 128.

- The highest value is 140.

- The lowest value is 96.

- $140 - 96 = 44$

- The range of this set of data is 44.

Example 6

Find the range of this set of data: 37, 44, 36, 29, 78, 15, 57, 54, 63, 27, and 48.

- The highest value is 78.

- The lowest value is 27.

- $78 - 27 = 51$

- The range of this set of data is 51.

Problems

Find the range of each set of data in problems 5 through 8.

> Outliers are numbers in a data set that are either much higher or much lower that the other numbers in the set.

Example 7

Find the outlier of this set of data: 88, 90 96, 93, 87, 12, 85, and 94.

- The outlier is 12.

Example 8

Find the outlier of this set of data: 67, 54, 49, 76, 64, 59, 60, 72, 123, 44, and 66.

- The outlier is 123.

Problems

Identify the outlier in each set of data.

9. 70, 77, 75, 68, 98, 70, 72, and 71.

10. 14, 22, 17, 61, 20, 16, and 15.

11. 1376, 1645, 1783, 1455, 3754, 1790, 1384, 1643, 1492, and 1776.

12. 62, 65, 93, 51, 55, 14, 79, 85, 55, 72, 78, 83, 91, and 76.

BOX PLOTS

Another way to display a distribution of one-variable numerical data is with a box plot. A box plot is the only display of data that clearly shows the median, quartiles, range, and outliers of a data set.

For additional information, see the Math Notes boxes in Lessons 8.1.4 and 8.1.5 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 9A materials.

Example 1

Display this data in a box plot:
51, 55, 55, 62, 65, 72, 76, 78, 79, 82, 83, 85, 91, and 93.

- Since this data is already in order from least to greatest, the range is $93 - 51 = 42$. Thus you start with a number line with equal intervals from 50 to 100.

- The median of the set of data is 77. A vertical segment is drawn at this value above the number line.

- The median of the lower half of the data (the lower quartile) is 62. A vertical segment is drawn at this value above the number line.

- The median of the upper half of the data (the upper quartile) is 83. A vertical segment is drawn at this value above the number line.

- A box is drawn between the upper and lower quartiles.

- Place a vertical segment at the minimum value (51) and at the maximum value (93). Use a line segment to connect the minimum to the box and the maximum to the box.
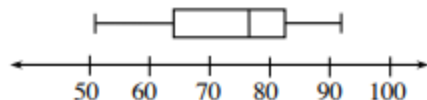
Example 2

Display this data in a box plot:
62, 65, 93, 51, 12, 79, 85, 55, 72, 78, 83, 91, and 76.

- Place the data in order from least to greatest: 12, 51, 55, 62, 65, 72, 76, 78, 79, 83, 85, 91, 93. The range is $93 - 12 = 81$. Thus you want a number line with equal intervals from 10 to 100.

- Find the median of the set of data: 76. Draw the line segment.

- Find the lower quartile: $55 + 62 = 117$; $117 \div 2 = 58.5$. Draw the line segment.

- Find the upper quartile: $83 + 85 = 168$; $168 \div 2 = 84$. Draw the line segment.

- Draw the box connecting the upper and lower quartiles. Place a line segment at the minimum value (12) and a line segment at the maximum value (93). Connect the minimum and maximum values to the box.
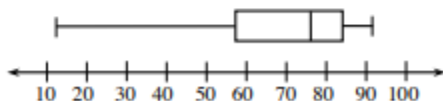
Problems

Create a stem-and-leaf plot and a box plot for each set of data in problems 1 through 4.

1. 45, 47, 52, 85, 46, 32, 83, 80, and 75.

2. 75, 62, 56, 80, 72, 55, 54, and 80.

3. 49, 54, 52, 58, 61, 72, 73, 78, 73, 82, 83, 73, 61, 67, and 68.

4. 65, 35, 48, 29, 57, 87, 94, 68, 86, 73, 58, 74, 85, 91, 88, and 97.

5. Given a set of data: 265, 263, 269, 259, 267, 264, 253, 275, 264, 260, 273, 257, and 291.

    a. Make a stem-and-leaf plot of this data.

    b. Find the mean and median of this data.

    c. Find the range of this data.

    d. Make a box plot for this data.

GRAPHICAL REPRESENTATIONS OF DATA   1.1.4

Students represent distributions of single-variable data numerical data using dot plots, stem-and-leaf plots, box plots, and histograms. They represent categorical one-variable data on bar graphs. Each representation communicates information in a slightly different way.

HISTOGRAMS AND BAR GRAPHS

Histograms and bar graphs are visual ways to represent data. Both consist of vertical bars (called bins) with heights that represent the number of data points (called the frequency) in each bin.

Histograms are for displaying distributions of numerical data. In a histogram each bar represents the number of data elements within a certain range of values. All the bars touch each other. Values at the left side of a bin's range are included in that bin. Each range of values should have the same width.

Bar graphs are for displaying categorical data. In a bar graph each bar represents the number of data elements in a certain category. All the bars are the same width and are separated from each other.

For additional information and examples, see the Math Notes boxes in Lessons 2.1.2 and 2.2.1 of the *Core Connections, Course 1* text. For additional examples and practice, see the *Core Connections, Course 1* Checkpoint 9A materials at the back of the text.
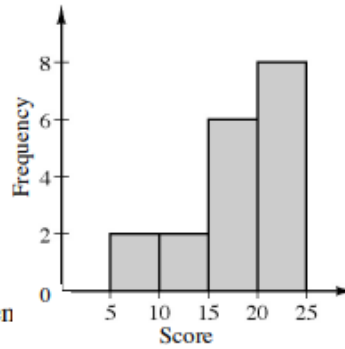
Example 1

The scores for a 25-point quiz are listed below arranged from least to greatest.

7, 7, 12, 13, 15, 16, 16, 16, 18, 19, 20, 20, 20, 21, 21, 22, 23, 24

Using intervals of five points, create a histogram for the class.

See histogram at right. Scores on the right end of the interval are included in the next interval. The interval between 10 and 15 only includes the two scores of 12 and 13. The interval between six scores of 15, 16, 16, 16, 18, and 19.

Example 2

Ms. Lim asked each of her students about their favorite kind of pet. Based on their responses, she drew the bar graph at right. Use the bar graph to answer each question.

   a. What is the favorite pet?

   b. How many students chose a bird as their favorite pet?

   c. What was the least favorite pet?

   d. If every student voted once, how many students are in the class?
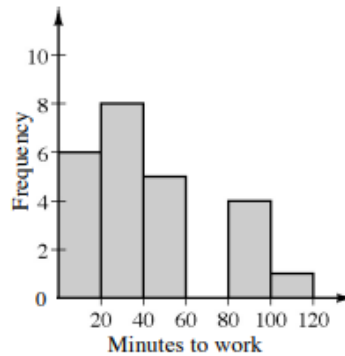
Answers: a. dog   b. 6   c. fish   d. 28
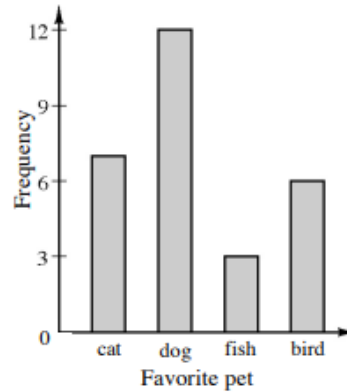
Problems

1.   Mr. Diaz surveyed his employees about the time it takes them to get to work. The results are shown in the histogram at right.

   a. How many employees completed the survey?

   b. How many employees get to work in less than 20 minutes?

   c. How many employees get to work in less than 40 minutes?

   d. How many employees take 60 minutes to get to work?

2. The two sixth grade classes at Vista Middle School voted for their favorite dessert. The results are shown in the bar graph at right for the five favorite choices.

a. What was the favorite dessert and how many students made that choice?

b. How many students selected cake as their favorite dessert?

c. How many students selected yogurt as their favorite?

d. How many more students selected ice cream than pudding?
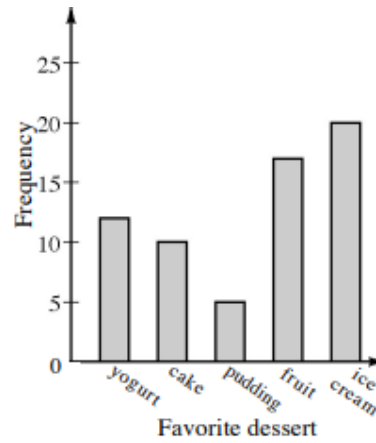
3. Mr. Fernandez asked 30 people at work how many pets they owned. The results are shown at right. Make a histogram to display this data. Use intervals of one pet.

0 pets	5 people
1 pet	8 people
2 pets	10 people
3 pets	3 people
4 pets	2 people
5 pets	1 person
9 pets	1 person

Answers

1. $m\angle a = 100°$ 2. $m\angle b = 55°$ 3. $m\angle c = 105°$
                                                  $m\angle d = 75°$
                                                  $m\angle e = 105°$

1. 113.04 cm^2 2. 32.15 in.^2 3. 200.96 ft^2 4. $\frac{11}{14} \text{ m}^2$

1. 314 mm^2 2. 192 cm^2 3. 210 ft^2

1. 12 ft^3 2. 168 cm^3 3. 240 in.^3

1. 30.57   2. $30.\overline{3}$ 3. $86.1\overline{3}$ 4. $106.\overline{6}$

5. median 30; range 8   6. median 28.5; range 9   7. median 82; range 47   8. median 107; range 27

9. 98   10. 61   11. 3754   12. 14

1.
```
3 | 2
4 | 5 6 7
5 | 2
7 | 5
8 | 0 3 5
```
box plot: 30 40 50 60 70 80 90

2.
```
5 | 4 5 6
6 | 2
7 | 2 5
8 | 0 0
```
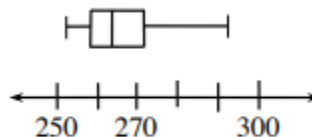box plot: 50 60 70 80

3.
```
4 | 9
5 | 2 4 8
6 | 1 1 7 8
7 | 2 3 3 3 8
8 | 2 3
```
box plot: 40 50 60 70 80 90

4.
```
2 | 9
3 | 5
4 | 8
5 | 7 8
6 | 5 8
7 | 3 4
8 | 5 6 7 8
9 | 1 4 7
```
box plot: 20 30 40 50 60 70 80 90 100
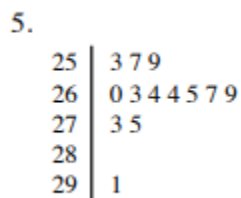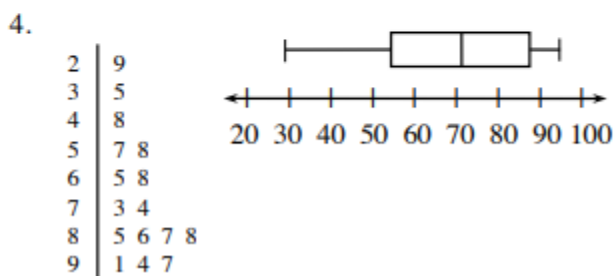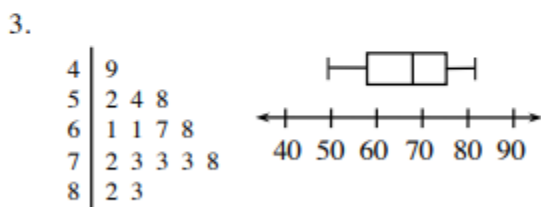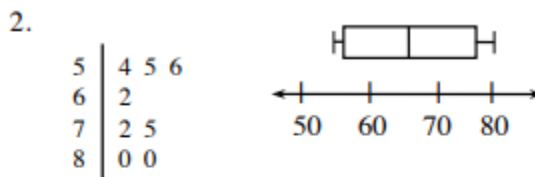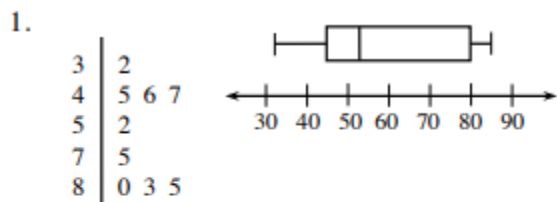
5.
```
25 | 3 7 9
26 | 0 3 4 4 5 7 9
27 | 3 5
28 |
29 | 1
```
Mean: 266.15
Median: 264
Range: 38

box plot: 250 270 300

1. a. 24   b. 6   c. 14   d. 0   2. a. ice cream 20   b. 10

                                                         c. 12   d. 15

3.

Frequency vs. Number of pets histogram